Research Statement

I am an algebraist with expertise in the areas of representation theory of Lie superalgebras, associative superalgebras, and algebraic objects related to mathematical physics. As a postdoctoral fellow, I have extended my research to include topics in enumerative and algebraic combinatorics.

My research is collaborative, and I welcome collaboration in familiar areas and in newer undertakings. The referenced open problems here, their subproblems, and other ideas in mind are suitable for undergraduate projects and theses, doctoral proposals, or scholarly contributions to academic journals.

In what follows, I provide a technical description of the motivation and history of my work in Lie superalgebras and super representation theory. This is then followed by a description of the work I am currently supervising and mentoring, in combinatorics, as I serve as the Postdoctoral Research Advisor for the Mathematical Sciences Research Institute's Undergraduate Program 2021.

1 Super Representation Theory

1.1 History

1.1.1 Superalgebras

In 1941, Whitehead defined a product on the graded homotopy groups of a pointed topological space, the first non-trivial example of a Lie superalgebra. Whitehead's work in algebraic topology [Whi41] was known to mathematicians who defined new graded geo-algebraic structures, such as $\mathbb{Z}_2$-graded algebras, or superalgebras to physicists, and their modules. A Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$ would come to be a $\mathbb{Z}_2$-graded vector space, even (respectively, odd) elements found in $\mathfrak{g}_{\bar{0}}$ (respectively, $\mathfrak{g}_{\bar{1}}$), with a parity-respecting bilinear multiplication termed the Lie superbracket $[\cdot,\cdot]$ inducing[1] a symmetric intertwining map $\mathfrak{g}_{\bar{1}} \otimes \mathfrak{g}_{\bar{1}} \to \mathfrak{g}_{\bar{0}}$ of $(\mathfrak{g}_{\bar{0}}, [\cdot,\cdot]\big|_{\mathfrak{g}_{\bar{0}}})$-modules. Researchers of the 1960s and 1970s furthered the systematic study of Lie superalgebras with a view for supermanifolds and the need for physicists to extend the classical symmetry formulation behind Wigner's Nobel Prize to one incorporating bosonic ("even") and fermionic ("odd") particles simultaneously—the famed supersymmetry (SUSY) [see FF77; Zum06]. In many regards, supermathematics is synonymous with the study of $\mathbb{Z}_2$-graded structures and spaces with Grassmann-valued coordinates. Now the algebraic development of supermathematics has become its own source of motivation—even providing a dictionary [FSS00]—beginning with the classification of simple finite-dimensional Lie superalgebras over algebraically closed fields of characteristic zero in [Kac75].

1.1.2 Orthosymplectic Lie Superalgebras

Even more, the investigation of $\mathbb{Z}_2$-graded non-associative algebras featuring orthogonal and symplectic parts, the so-called orthosymplectic Lie superalgebras, proves to be quite fruitful within and outside of mathematics. Specifically, the Lie superalgebra $\mathfrak{osp}(1|2n,\mathbb{R})$ is of great importance to superconformal theories; see Chapters 6 and 7 of [Far84] for an early survey of physical applications. In mathematics, the natural question to describe simple objects in a module category of $\mathfrak{osp}(1|2n,\mathbb{C})$, and of other classical Lie superalgebras defined in [Kac77], was considered by Dimitrov, Mathieu, Penkov in [DMP00]. Gorelik and Grantcharov completed the classification

[1] We also require $[x,[x,x]] = 0,\ \forall x \in \mathfrak{g}_{\bar{1}}$.

began in [DMP00] by publishing [FGG16] then [GG18], the former with Ferguson, who classified the simple bounded highest-weight modules of $\mathfrak{osp}(1|2n,\mathbb{C})$ in his thesis [Fer15]. Primitive vectors in certain tensor product representations of $\mathfrak{osp}(1|2n,\mathbb{C})$ were essential to the completion of [FGG16]. Coulembier also paid close attention to primitive vectors in [Cou13] to determine whether tensor products of certain irreducible highest-weight $\mathfrak{osp}(2m+1|2n,\mathbb{C})$-representations were completely reducible; a series of papers by Coulembier and co-authors [Cou10; Cou13; CSS14] motivated inspecting the reducibility of certain $\mathfrak{osp}(1|2n,\mathbb{R})$-modules to establishing a Clifford analysis on supermanifolds.

In [Wil20] I described explicit bases of irreducible summands of tensor product representations formed by tensoring polynomials $\mathbb{C}[x_1,\dots,x_n]$ with the standard representation $\mathbb{C}^{1|2n}$. I also established a map of superalgebras from $U(\mathfrak{osp}(1|2n,\mathbb{C}))$ to the Clifford/Weyl superalgebra $A^+_{n|2n}$ as studied in [HS18]. My work provides a family of infinite-dimensional representations. The infinite-dimensional representation theory of $\mathfrak{osp}(1|2n)$ over the field of real or complex numbers presents a wealth of open problems that I continue to address.

1.1.3 Super Reduction algebras

Localized step algebras or reduction algebras [Zhe94; Mic73], also known as transvector algebras [DER17] and symmetry algebras [Zhe97], have been an object of study with much progress made by Khoroshkin, Ogievetsky, et al. [KO08; KO14]. The general construction of reduction algebras extends to super, quantum, and affine cases [AM15; MM14; van75; Zhe89] and the embedding of the reductive algebra $\mathfrak{g}$ into the larger associative algebra U gives rise to types of reduction algebras. In [KO17], a main result is the complete presentation of generators and relations for the diagonal reduction algebra for $\mathfrak{gl}(n)$. Determining complete presentations of diagonal reduction superalgebras connected to Lie superalgebras is an open problems in the study of associative superalgebras and super representation theory. Hartwig and I [HW21] established a complete presentation of the diagonal reduction algebra of $\mathfrak{osp}(1|2,\mathbb{C})$ in terms of generators and relations, establishing groundwork for further representation theoretic applications and categorical considerations on the structure of diagonal reduction algebras associated to Lie superalgebras. My interests in reduction algebras is distinct from my PhD work. Still, I have been able to find applications to my previous results in order to produce fresh research questions and results.

1.2 Research Results

My current work addresses multiple directions within the study of superalgebras. I emphasize that my written results are found in [Wil20; HW21; GW], which I use to cite some of my main theorems below.

1.2.1 Oscillator Representations and Tensor Product Representations

Throughout the following paragraphs, the ground field is $\mathbb{C}$. Consider $\mathbb{C}^{1+2n}$ as a $\mathbb{Z}_2$-graded vector space (super vector space):

$$\mathbb{C}^{1|2n} = (\mathbb{C}^{1|2n})_{\bar{0}} \oplus (\mathbb{C}^{1|2n})_{\bar{1}} = \mathbb{C}v_0 \oplus \left(\bigoplus_{i>0} \mathbb{C}v_i\right),$$

where $v_i = e_{i+1}$, $0 \leq i \leq 2n$, are the standard basis vectors of $\mathbb{C}^{1+2n}$. Define the Lie superalgebra $\mathfrak{osp}(1|2n)$ as the set of endomorphisms on $\mathbb{C}^{1|2n}$ preserving the form represented by the matrix

$$J_n = \overset{(1+2n)\times(1+2n)}{\left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 0 & -I_n \\ 0 & I_n & 0 \end{array}\right]}$$

with the Lie superbracket on any pair of homogeneous morphisms[2] X and Y:

$$[X, Y] = XY - (-1)^{|X||Y|}YX.$$

A motivated goal of my work in [GW] is to create a map, on the level of $\mathfrak{osp}(1|2n)$-representations, between $\mathbb{C}[x_1, \ldots, x_n] \otimes \mathbb{C}[\eta_1, \ldots, \eta_{2n}]$ and $\mathbb{C}[x_1, \ldots, x_n] \otimes \mathbb{C}[\eta_0; \eta_1, \ldots, \eta_{2n}]$, where η_0 is an even variable which does not square to 0 and η_i is odd with $\eta_i^2 = 0, 1 \leq i \leq 2n$. The right factor $\mathbb{C}[\eta_1, \ldots, \eta_{2n}]$ of the first tensor product corresponds to tensoring the usual polynomials in n variables with exterior powers. I built from my theses to establish a next step in answering the problem of defining a map between certain tensor-product representations of $\mathfrak{osp}(1|2n)$.

Theorem 1.1 ([GW]). *The Lie superalgebra $\mathfrak{osp}(1|2n)$ acts as commuting and anti-commuting super differential operators. Formally, the following correspondence defines a homomorphism $\Omega : U(\mathfrak{osp}(1|2n)) \to \mathrm{End}(\mathbb{C}[\eta_0; \eta_1, \ldots, \eta_{2n}])$ of associative superalgebras with identity, for $i \neq j$:*

$$\begin{aligned}
X_{\delta_i-\delta_j} &\longmapsto \eta_i\partial_{\eta_j} - \eta_{j+n}\partial_{\eta_{i+n}};\\
X_{2\delta_i} &\longmapsto \eta_i\partial_{\eta_{i+n}};\\
X_{-2\delta_i} &\longmapsto \eta_{i+n}\partial_{\eta_i};\\
X_{\delta_i+\delta_j} &\longmapsto \eta_i\partial_{\eta_{j+n}} + \eta_j\partial_{\eta_{i+n}};\\
X_{-\delta_i-\delta_j} &\longmapsto \eta_{i+n}\partial_{\eta_j} + \eta_{j+n}\partial_{\eta_i};\\
h_{\delta_i-\delta_j} &\longmapsto \eta_i\partial_{\eta_i} - \eta_j\partial_{\eta_j} + \eta_{j+n}\partial_{\eta_{j+n}} - \eta_{i+n}\partial_{\eta_{i+n}};\\
h_{2\delta_i} &\longmapsto \eta_i\partial_{\eta_i} - \eta_{i+n}\partial_{\eta_{i+n}};\\
X_{\delta_i} &\longmapsto \eta_0\partial_{\eta_{i+n}} + \eta_i\partial_{\eta_0};\\
X_{-\delta_i} &\longmapsto \eta_0\partial_{\eta_i} - \eta_{i+n}\partial_{\eta_0}.
\end{aligned}$$

Corollary 1.2 ([GW]). *The space of polynomials $\mathbb{C}[\eta_0; \eta_1, \ldots, \eta_{2n}]$ in the commuting variable η_0 and anti-commuting and nilpotent variables η_i, for $1 \leq i \leq 2n$, is a representation of $\mathfrak{osp}(1|2n)$.*

The literature [Fau10; Cou10; DER17] supports studying the relation between the super vector spaces in my results above and super analogues of harmonic functions and Clifford-analytic functions. I welcome joint work in these areas with colleagues and in supervision of the interested student.

1.2.2 Weyl Reduction Algebra

Extending my work in [HW21] is a natural pursuit. That is, I am working to determine the structure of the diagonal reduction algebra of $\mathfrak{osp}(1|2n)$, for $n > 1$, by exhibiting a complete presentation in terms of generators and relations. Another question is to determine a clear highest weight theory for the reduction algebra $Z(\mathfrak{osp}(1|2) \times \mathfrak{osp}(1|2), \mathfrak{osp}(1|2))$. More of my

[2] Every pair of objects in the category of complex super vector spaces $\mathbf{SVect}_{\mathbb{C}}$ has an internal Hom; that is, there is a natural grading on morphisms corresponding to parity-preserving and parity-reversing maps.

ongoing work is in the development of the *super Weyl reduction algebra* using a composition of the diagonal embedding and the correspondence between $U(\mathfrak{osp}(1|2n))$ and the algebra of polynomial differential operators [see any of Mus99; Fer15; Wil20]. Similarly, the correspondence I found in Theorem 1.1 provides another embedding of $\mathfrak{osp}(1|2n)$ into an associative superalgebra containing $U(\mathfrak{osp}(1|2n))$, a first step in defining reduction algebras of new types.

More precisely, let $\mathcal{D}(n)$ be the nth Weyl superalgebra. Then $U(\mathfrak{osp}(1|2n)) \otimes \mathcal{D}(n)$ is an associative superalgebra containing $U(\mathfrak{osp}(1|2n))$ that allows us to define a reduction algebra whose presentation and representation theory are compatible with the study of the infinite-dimensional representation theory featured in my work. See Chapter 7 of [Wil20] for a powerful correspondences between associative superalgebras when regarding the goal to understand infinite-dimensional representation theory.

Another part of my research undertakings are influenced by a conversation with Serganova and involve the functorial nature of reduction algebras. More precisely, I am working to answer the following questions:

1. How does the diagonal reduction algebra of $\mathfrak{sp}(2n)$ relate to the diagonal reduction algebra of $\mathfrak{osp}(1|2n)$?
2. How is the diagonal reduction algebra Z of a Lie superalgebra $\mathfrak{L}$ related to the diagonal reduction algebra of the Lie algebra $\mathfrak{L}_{\overline{0}}$, which is the even part of $\mathfrak{L}$?

I remark that the above two questions are general enough that each could serve as a strong basis for a PhD thesis under my supervision.

2 Combinatorics

As previously mentioned, I am the Mathematical Sciences Research Institute Undergraduate Program 2021 Postdoctoral Research Advisor. In this capacity, I serve as a research mentor to 18 undergraduate mathematicians from underrepresented groups.[3] The area of study is in combinatorics, particuarly, in generalizaed parking functions. My research questions focus on interval parking functions, distance parking functions, tiered/color parking functions, zone and preferential parking functions, rational parking functions, and I make connections between parking functions and the Tower of Hanoi. Below I provide a short introduction to parking functions.

2.1 Parking Functions

The “parking" problem [KW66] related to hashing and the storage of data is the source of many modern questions in combinatorics. An overview of recent projects is found in [Car+20].

For any natural number n, let $[n]$ be defined as the set $\{1, 2, \dots, n\}$. A parking function of length n can be defined as a vector $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ in $[n]^n$ such that there exists a permutation σ in $\mathfrak{S}_n$ by which $\alpha_{\sigma(i)} \leq i$, for all i in $[n]$. In particular, a vector α is a parking function if its non-decreasing rearrangement satisfies the condition that the value of each entry is less than or equal to the index of the entry. Furthermore, any permutation of a parking function is a parking function itself; hence, all permutations on $[n]$ are parking functions of length n.

The associated visualization is of a set of n cars, say, $\{c_1, c_2, \dots, c_n\}$, attempting to park in n parking spots. The parking spots are naturally ordered on one side of a one-way street, and the cars park according to a preference vector $p = \{p_1, p_2, \dots, p_n\}$ in $[n]^n$, whereby car c_i will look to park in spot p_i initially. If parking spot p_i is occupied, then car c_i will move forward to

[3] One may say minoritized students along racial and ethnic axes, but in this document I will use terminology of the program.

the next open spot algorithmically without being able to move backwards or cycle to return to the beginning of the street. If all cars can park, then the preference vector p is called a parking function.

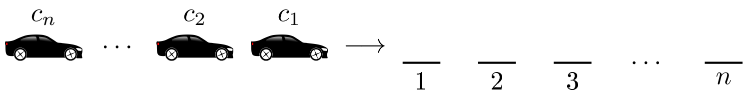

Figure 1: Parking function illustration.

The number of parking functions of length n is known to be $(n+1)^{n-1}$, [KW66] however, there are many other statistics [Col+20], interesting bijections and results on enumeration[Cam+08; Had21], algebraic structures and group actions[NT03; BER11], and modified definitions [ARR14; Duk21] that fill the literature. The breadth and depth of the study of parking functions allows one to involve students in the research process at all stages. Given my expertise in algebra, I can use algebraic techniques to extend the study to applications in representation theory and invariant theory, among other active algebraic research areas. My expertise in superalgebras and experience in advising makes me willing and capable to lead students in topics ripe for future research both at the undergraduate and graduate levels.

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