Representations_of__1_2n_ (1)

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$\mathfrak{g}$: Lie alg

V : $\mathfrak{g}$-module

$V^+ \subseteq V$: space of primitive vectors

$$V = \bigoplus_{i=1}^{\text{all primitive vectors}} U(\mathfrak{g}_-) w_i, \quad w_i \in V^+$$

$$x \in \mathfrak{g} \to ad_x : \mathfrak{G} \to \mathfrak{G}$$
$$y \longmapsto [x, y]$$

- $\mathfrak{G}$: Lie superalgebra $\mathfrak{osp}(1|2) \times \mathfrak{osp}(1|2)$
- δ: $\mathfrak{osp}(1|2) \to \mathfrak{G}$, $\quad x \mapsto (x, x)$ — diagonal embedding
- $\mathfrak{g}$: reductive image of $\mathfrak{osp}(1|2)$ under δ
- $\mathfrak{p}$: reductive complement of $\mathfrak{g}$: $\quad (\mathfrak{p} \oplus \mathfrak{g} \overset{\mathfrak{g}\text{-modules}}{=} \mathfrak{G})$
- $\mathfrak{H}$: Cartan sublagebra of $\mathfrak{g}$: $\quad \mathbb{C}H$

$$osp(1|2) \text{ “}=\text{” } \underbrace{sl(2)}_{3\text{-dim}} \oplus \underbrace{\overset{2\text{-dim}}{sl(2)\text{-module}}}_{2\text{-dim}}$$

$x_{-2\alpha}$	$x_{-\alpha}$
h	x_{α}
$x_{2\alpha}$	

- $R = D^{-1}U(\mathfrak{H})$ — ring of dynamical scalars
- $D^{-1}U(\mathfrak{H}) = \mathbb{C}[H][(H-n)^{-1} \mid n \in \mathbb{Z}]$

$$U(h) = \mathbb{C}[H]$$

$$U(sl(2)) = \{ f^j h^k e^\ell \mid \{f, e, h\} \text{ is a basis for } sl(2), \ j, k, \ell \in \mathbb{N}_0 \}$$

[illegible]

basis

$eh \longrightarrow \boxed{he - 2e}$

$[h,e] = 2e$

$he - eh = 2e$

$$\begin{tikzcd} \mathfrak{g} \arrow[r, "\text{Lie alg.}"] \arrow[dr] & B \\ & U(\mathfrak{g}) \arrow[u, dashed, "\text{assoc. algebras}"'] \end{tikzcd}$$

$U(osp(1|2))$: basis $= \{ x_{-2\alpha}^{j} x_{-\alpha}^{p} h^{k} x_{\alpha}^{q} x_{2\alpha}^{\ell} \mid j,k,\ell \in \mathbb{N}_0,\ p,q \in \{0,1\} \}$

$x_{-2\alpha}, x_{-\alpha}, h, x_{\alpha}, x_{2\alpha}$

$-2x_{2\alpha} = [x_\alpha, x_\alpha] = x_\alpha x_\alpha + x_\alpha x_\alpha = 2x_\alpha^2$

$-x_{2\alpha} = x_\alpha^2$

$R = D^{-1}U(\mathfrak{h})$

$\frac{1}{H-1},\ \frac{1}{(H+2)(H-1)},\ \frac{H-2}{H+1}$

Extension of scalars

$$osp(1|2) \xrightarrow{\delta} \underbrace{osp(1|2) \times osp(1|2)}_{\delta(osp(1|2)) = \mathfrak{g} \subseteq G} \longrightarrow U(G) = U(osp(1|2)) \otimes U(osp(1|2)) \longrightarrow R \otimes_{U(\mathfrak{h})} U(G) = U$$

V, W: finite-dim

$\dim(\text{direct sum } V \oplus W) = \dim(V) + \dim(W)$

$\dim(\text{tensor product } V \otimes W) = \dim(V)\dim(W)$

basis of $V \otimes W$: $\{ v_i \otimes w_j \mid i = 1,2,\dots,\dim(V),\ j = 1,2,\dots,\dim(W) \}$

Basis elements in V basis elements in W

Super case: $$V \otimes W = (V_{\bar{0}} \otimes W_{\bar{0}}) \oplus (V_{\bar{1}} \otimes W_{\bar{1}}) \quad \text{] even part } (V \otimes W)_{\bar{0}}$$

$V = V_{\bar{0}} \oplus V_{\bar{1}}$ $$\oplus (V_{\bar{0}} \otimes W_{\bar{1}}) \oplus (V_{\bar{1}} \otimes W_{\bar{0}}) \quad \text{] odd part } (V \otimes W)_{\bar{1}}$$

$W = W_{\bar{0}} \oplus W_{\bar{1}}$