

Representations_of__1_2n_(1)



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\mathfrak{g} : Lie alg

V : \mathfrak{g} -module

$V^+ \subseteq V$: space of primitive vectors

$$V = \bigoplus_{i=1}^{\text{all primitive vectors}} U(\mathfrak{g}) w_i, \quad w_i \in V^+$$

$$x \in \mathfrak{g} \rightarrow \text{ad}_x: \mathfrak{g} \rightarrow \mathfrak{g}$$

$$y \mapsto [x, y]$$

- \mathfrak{G} : Lie superalgebra $\mathfrak{osp}(1|2) \times \mathfrak{osp}(1|2)$
- $\delta: \mathfrak{osp}(1|2) \rightarrow \mathfrak{G}, \quad x \mapsto (x, x)$
- \mathfrak{g} : reductive image of $\mathfrak{osp}(1|2)$ under δ
- \mathfrak{p} : reductive complement of \mathfrak{g} : $(\mathfrak{p} \oplus \mathfrak{g})^{\mathfrak{g}\text{-modules}} = \mathfrak{G}$
- \mathfrak{h} : Cartan subalgebra of \mathfrak{g} : $\mathbb{C}H$

diagonal embedding

$$\mathfrak{osp}(1|2) \stackrel{\cong}{=} \mathfrak{sl}(2) \oplus \mathfrak{sl}(2)\text{-module}$$

$\mathfrak{sl}(2)$
 \swarrow
 3-dim
 $x_{-2\alpha}$
 h
 $x_{2\alpha}$

$\mathfrak{sl}(2)\text{-module}$
 \swarrow
 2-dim
 $x_{-\alpha}$
 x_{α}

- $R = D^{-1}U(\mathfrak{h})$
- $D^{-1}U(\mathfrak{h}) = \mathbb{C}[H][(H - n)^{-1} \mid n \in \mathbb{Z}]$

ring of dynamical scalars

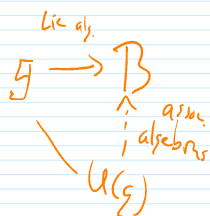
$$U(\mathfrak{h}) = \mathbb{C}[H]$$

$$U(\mathfrak{sl}(2)) = \{ f^j h^k e^{\pm l} \mid \{f, e, h\} \text{ is a basis for } \mathfrak{sl}(2), j, k, l \in \mathbb{N}_0 \}$$



$U(\mathfrak{sl}_2) = \langle \dots \rangle$

basis
 $e, h \rightarrow \boxed{he - 2e}$
 $[h, e] = 2e$
 $he - eh = 2e$



$U(\mathfrak{osp}(1|2))$: basis = $\{ x_{-2\alpha}^j x_{-\alpha}^p h^k x_{\alpha}^q x_{2\alpha}^r \mid j, k, r \in \mathbb{N}_0, p, q \in \{0, 1\} \}$

$x_{-2\alpha}, x_{-\alpha}, h, x_{\alpha}, x_{2\alpha}$

$-2x_{2\alpha} = [x_{\alpha}, x_{\alpha}] = x_{\alpha}x_{\alpha} + x_{\alpha}x_{\alpha} = 2x_{\alpha}^2$

$-x_{2\alpha} = x_{\alpha}^2$

$\mathcal{R} = D^{-1}U(\mathfrak{h})$

$\frac{1}{H-1}, \frac{1}{(H+2)(H-1)}, \frac{H-2}{H+1}$

Extension & scalars

$\mathfrak{osp}(1|2) \xrightarrow{\mathfrak{d}} \underbrace{\mathfrak{osp}(1|2) \times \mathfrak{osp}(1|2)}_{\mathfrak{d}(\mathfrak{osp}(1|2)) = \mathfrak{g} \subseteq \mathfrak{G}} \rightarrow U(\mathfrak{G}) = U(\mathfrak{osp}(1|2)) \otimes U(\mathfrak{osp}(1|2)) \rightarrow \mathcal{R} \otimes_{U(\mathfrak{h})} U(\mathfrak{G}) = U$

V, W : finite-dim

$\dim(\text{direct sum } V \oplus W) = \dim(V) + \dim(W)$

$\dim(\text{tensor product } V \otimes W) = \dim(V)\dim(W)$

basis of $V \otimes W$: $\{v_i \otimes w_j \mid i = 1, 2, \dots, \dim(V), j = 1, 2, \dots, \dim(W)\}$

even elements in V odd elements in W

Super case:

$$V = V_0 \oplus V_1$$

$$W = W_0 \oplus W_1$$

$$V \otimes W = (V_0 \otimes W_0) \oplus (V_1 \otimes W_1) \quad] \text{ even part } (V \otimes W)_0 \\ \oplus (V_0 \otimes W_1) \oplus (V_1 \otimes W_0) \quad] \text{ odd part } (V \otimes W)_1$$