# Title may not apply

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Joint work with Jonas T. Hartwig

# Goals

- To show y'all some pictures
- To welcome you to study reduction algebras
- To share that I'll be at

  Morgan State University (Fall 2023)

# Reduction algebras recipe

1 associative algebra $A$

1 $A$-module $V$

1 system of equations $\varepsilon \cdot V = 0$ $(\varepsilon \subseteq A)$

generate the left ideal $A\varepsilon$

form the quotient $? / A\varepsilon$

An answer

$$N_A(A_\varepsilon)$$

The normalizer of $A_\varepsilon$ in $A$

If $v \in V$ is a solution to our system of equations

$$\varepsilon \cdot v = 0$$

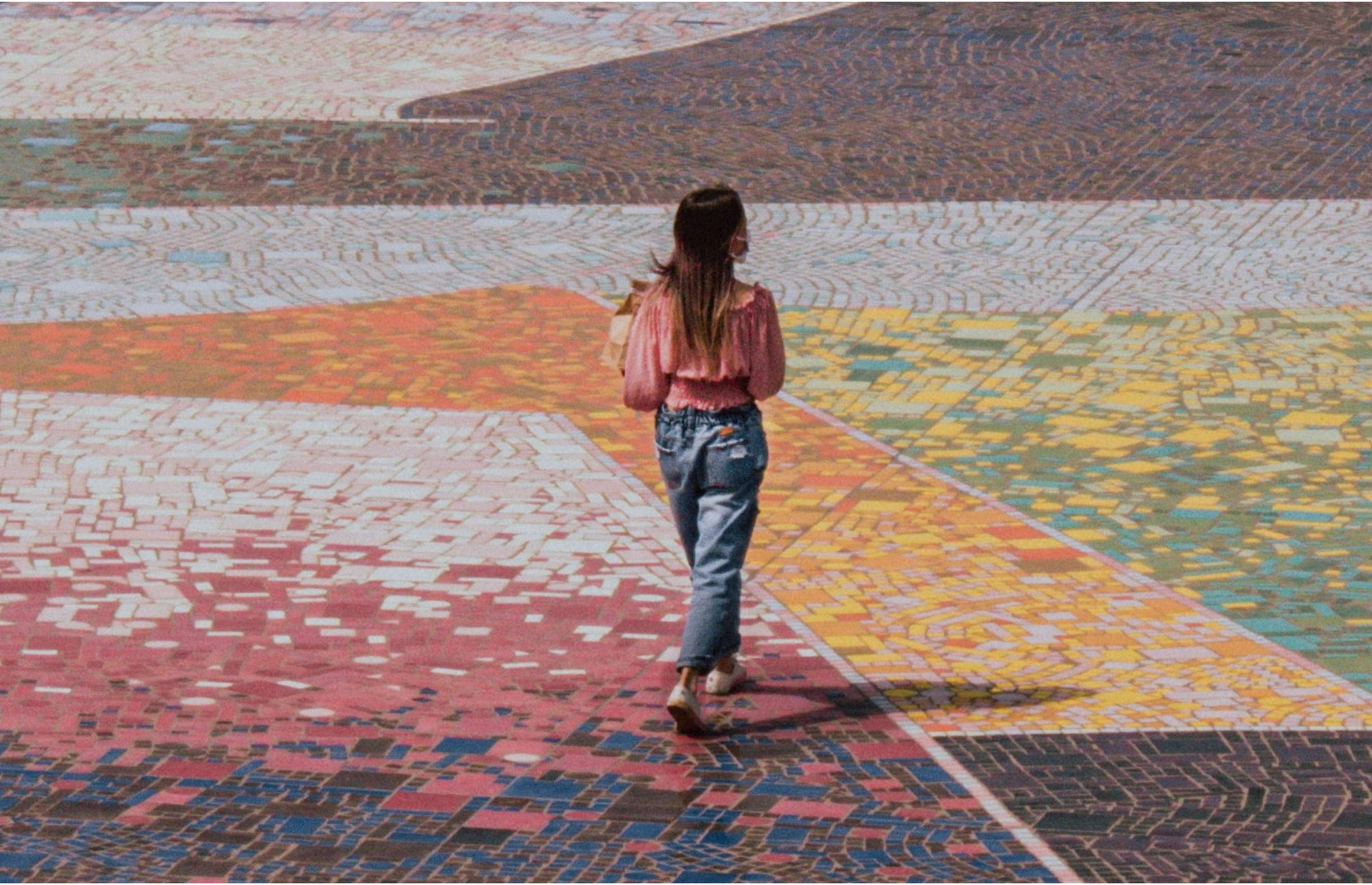
If $n \in N_A(A_\varepsilon)$, then

$$\varepsilon \cdot (n \cdot v) = 0$$

We just cooked up an algebra

$$Z = N_A(A_\varepsilon) / A_\varepsilon$$

$Z$ acts on solutions to our system of equations



"Painters or the painted"

Mickelsson – reduction problem

What if $\varepsilon \cdot V = 0 \to$ primitive vectors

$$\bar{V} = \bigoplus_{\{w \mid \varepsilon \cdot w = 0\}} U(\mathfrak{g}_-) w$$

Zhelobenko – extremal equations

What if $\varepsilon \cdot V = 0 \to$ physics

copy & $\mathfrak{sl}(2) \subseteq A_n$

Khoroshkin & Ogievetsky – diagonal reduction algebra

What if $\varepsilon \cdot V = 0 \to$ primitive vectors & $\otimes$-products

$$\mathfrak{g} \subseteq \mathfrak{g} \times \mathfrak{g}$$

painting with

Jonas T. Hartwig

Irmak Balçık

diagonal reduction algebra for osp(1|2)
https://doi.org/10.1134/S0040577922020015

Pomona College thesis

ghost center and representations of the diagonal reduction algebra of osp(1|2)
https://doi.org/10.1016/j.geomphys.2023.104788

## Zooming in!

Hartwig & W.
- introduced diagonal reduction superalgebra
- classified finite-dim irreps
- continuing: (semi-) differential reduction algebras

Bukey — $\underbrace{\text{solve for harmonic poly using } Z}_{\text{in spirit of Zhelobenko with computation}}$

EXTREMAL PROJECTOR — Tolstoy

DOUBLE COSET SPACE — diamond product

How do we determine $N_A(A\varepsilon)$?

Bigger picture? Back to the ingredients

$$\mathfrak{osp}(1|2n) = \mathfrak{g}_{\bar 0} \oplus \mathfrak{g}_{\bar 1} \xrightarrow[\text{LSAs}]{\text{map to}} A_n$$

$\mathfrak{g}_{\bar 0} \cong \mathfrak{sp}(2n)$

$\mathfrak{osp}(1|2n) \longrightarrow \mathfrak{osp}(1|2n) \times \mathfrak{osp}(1|2)$ (diagonal reduction algebra)

$\searrow A_n$

semi-differential reduction algebra

$$\underbrace{U(sp(4))}_{\text{rank } 2} \to A_2$$

positive roots

$\{\alpha, \beta+\alpha, \beta+2\alpha, \beta\}$

copy of $sp(4) \subseteq A_2 \otimes U(sp(4)) = A$

$sp(4) \hookrightarrow sp(4) \times sp(4) \hookrightarrow U(sp(4)) \otimes U(sp(4)) \hookrightarrow A_2 \otimes U(sp(4))$

$\mathcal{E}$ = positive nilpotent part of $A$ $(A_+)$

with respect to $A = A_- \oplus H \oplus A_+$ triangular decomposition from $sp(4) \hookrightarrow A$

$\mathcal{V} \in$ category of $sp(4)_+$ - locally-nilpotent $A$-modules think $\otimes$-modules of $sp(4)$

How do we cook?

How do we paint?

What are we doing?

Continuing motivation:

Decompose modules such as

$$\mathbb{C}[x_1, x_2] \otimes \mathbb{C}^{1|4} \quad \checkmark$$

$$\mathbb{C}[x_1, x_2] \otimes W$$

extremal projector for $n > 2$
computations

Irreps & reduction algebra when $A = A_n \otimes U(\mathfrak{osp}(1|2n))$

Categorical connections: $\mathfrak{h} \to \mathfrak{g}$

$$DRA(\mathfrak{h}) \to DRA(\mathfrak{g}) \to SDRA(\mathfrak{g})$$

# NEW CANVAS (MORE PAINTERS)

Dimitar Grantcharov


Saber Ahmed


Q-type reduction superalgebras